Advanced Network Analysis

Temporal ERGMs

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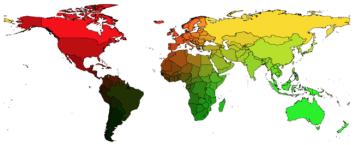
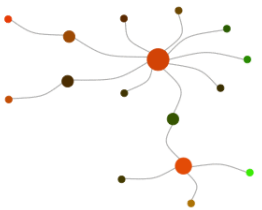
Longitudinal networks



-1965



Longitudinal networks

- Networks that change over time.
- Examples: networks of friends, conflict networks, trade networks.
- Want to model network dynamics within and across time periods.

Outline

- Setting up longitudinal network data
- Visualizing longitudinal data
- Descriptive statistics
- Inferential analysis

Getting the data ready

We are used to data in this format:

```
#install_github("ochyzh/networkdata")
library(networkdata)
data(allyData)
head(dyadData)[,1:8]
```

```
##   cname1 cname2 year ally war contiguity  id1_cinc  id2_cinc
## 2    CAN    USA 1991    1   0          1 0.0119571 0.1364806
## 3    MEX    USA 1991    1   0          1 0.0125758 0.1364806
## 4    COL    USA 1991    1   0          0 0.0046681 0.1364806
## 5    VEN    USA 1991    1   0          0 0.0052502 0.1364806
## 6    PER    USA 1991    1   0          0 0.0033841 0.1364806
## 7    BRA    USA 1991    1   0          0 0.0240151 0.1364806
```

Getting the data ready

We need to convert this information such that:

- the dependent variable must be a `list` of `network` objects
- nodal covariates are vertex attributes in the `list` of `network` objects
- dyadic covariates are included separately in a `list` of `matrices`

Start with setting up war

Output should look like this:

```
class(war)
```

```
## [1] "list"
```

```
length(war)
```

```
## [1] 10
```

```
class(war[[1]])
```

```
## [1] "matrix" "array"
```

```
dim(war[[1]])
```

```
## [1] 50 50
```

```
war[[1]][1:3,1:3]
```

```
##     USA CHN IND
## USA   0   0   0
## CHN   0   0   1
## IND   0   0   0
```

contiguity should be easier

Output should look like this:

```
class(contiguity)
```

```
## [1] "matrix" "array"
```

```
dim(contiguity)
```

```
## [1] 50 50
```

```
contiguity[1:3,1:3]
```

```
##     USA CHN IND
## USA   0   0   0
## CHN   0   0   1
## IND   0   1   0
```

Now set up DV with vertex attributes

Output should look like this:

```
class(ally)
```

```
## [1] "list"
```

```
length(ally)
```

```
## [1] 10
```

```
class(ally[[1]])
```

```
## [1] "network"
```

```
list.vertex.attributes(ally[[1]])
```

```
## [1] "cinc"         "cname"        "polity"       "vertex.names" "year"
```

Exploring temporal network data

The `statnet` package includes a range of "sub-packages" that enable you to understand the characteristics of dynamic networks:

- `networkDynamic`: storage and management of temporal network data
- `tsna`: descriptive statistics and graphics for exploratory network analysis
- `ndtv`: utilities for plotting temporal networks (including network movies)

Prepping data

First step is going to be formatting our list of `network` objects into a format that these packages can recognize:

```
allyDyn = networkDynamic(network.list=ally)
```

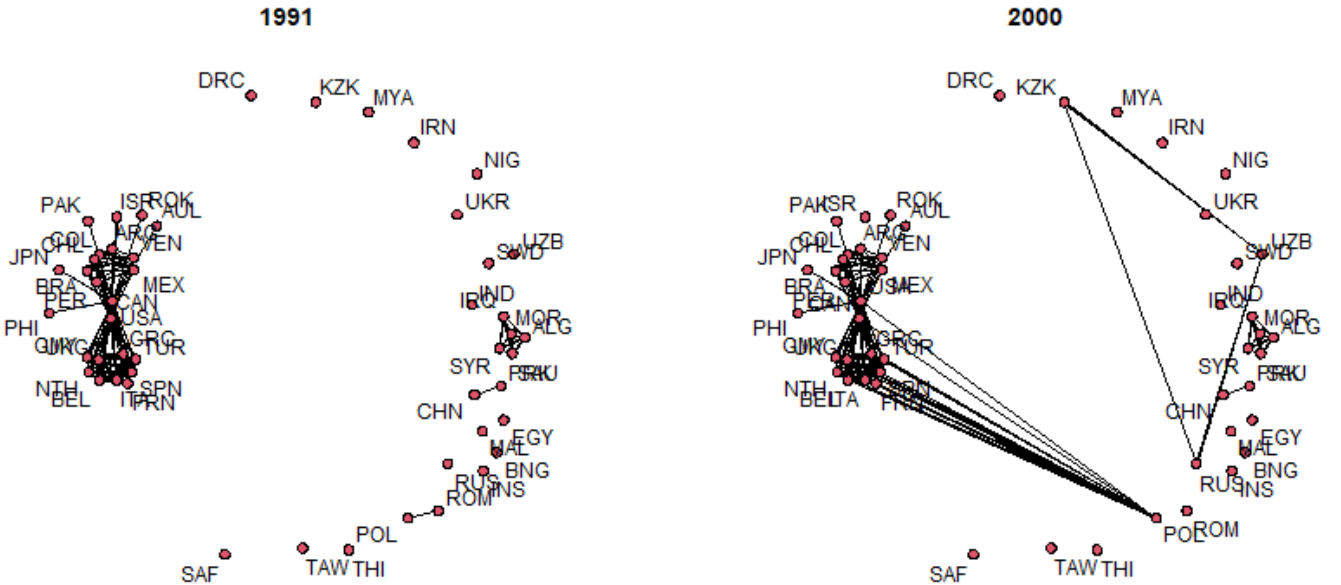
```
## Neither start or onsets specified, assuming start=0
## Onsets and termini not specified, assuming each network in network.list sh
## Argument base.net not specified, using first element of network.list inste
## Created net.obs.period to describe network
##  Network observation period info:
##   Number of observation spells: 1
##   Maximal time range observed: 0 until 10
##   Temporal mode: discrete
##   Time unit: step
##   Suggested time increment: 1
```

Quick snapshots

`networkDynamic` makes it easy to generate some snapshots of a longitudinal network:

```
par(mfrow = c(1,2))
p<-plot(
    network.extract(allyDyn, at = 0),
    main = "1991", displaylabels = T)
plot(
    network.extract(allyDyn, at = 9),
    main = "2000", displaylabels = T,coord=p)
```

Quick snapshots



Quick movie

`ndtv` makes it pretty easy to render a simple D3 movie for a longitudinal network:

```
library(ndtv)
render.d3movie(allyDyn,
    plot.par=list(displaylabels=T),filename="AlliesNetwork.html", lau
```

Descriptive analyses

`tsna` enables us to quickly calculate some basic descriptive statistics such as the density of a graph:

```
library(tsna)
tSnaStats(allyDyn, "gden") # Changes in graph density
```

```
## Time Series:
## Start = 0
## End = 10
## Frequency = 1
##          Series 1
##  [1,] 0.08816327
##  [2,] 0.09061224
##  [3,] 0.09061224
##  [4,] 0.08979592
##  [5,] 0.08979592
##  [6,] 0.08979592
##  [7,] 0.09795918
##  [8,] 0.09795918
##  [9,] 0.09795918
## [10,] 0.09795918
## [11,]         NA
```

Descriptive analyses

Can also examine changes in transitivity over time:

```
tSnaStats(allyDyn, "gtrans") # Changes in graph transitivity
```

```
## Time Series:
## Start = 0
## End = 10
## Frequency = 1
##         Series 1
##  [1,] 0.7617647
##  [2,] 0.7768924
##  [3,] 0.7768924
##  [4,] 0.7768924
##  [5,] 0.7768924
##  [6,] 0.7768924
##  [7,] 0.7974684
##  [8,] 0.7974684
##  [9,] 0.7974684
## [10,] 0.7974684
## [11,]        NA
```

Descriptive analyses

The `tErgmStats` enables us to calculate changes in `ergm` terms over time:

```
tErgmStats(allyDyn, "~ edges+triangle")
```

```
## Time Series:
## Start = 0
## End = 10
## Frequency = 1
##    edges triangle
##  0   108      259
##  1   111      260
##  2   111      260
##  3   110      260
##  4   110      260
##  5   110      260
##  6   120      315
##  7   120      315
##  8   120      315
##  9   120      315
## 10     0        0
```

Your Turn

1. For this exercise, you will work with the friendship data from `library(RSiena)`. Run the following code to load the data.

```
library(RSiena)
friend.data.w1 <- s501
friend.data.w2 <- s502
friend.data.w3 <- s503
drink <- s50a
smoke <- s50s
```

1. Format the friendship data from as a `networkDynamic` object.
2. Summarize changes in friendships (edges), triangles, and number of nodes with indegrees 1 nd 2, over time.
3. Plot the first and the third waves of the friends network side-by-side using the `network.extract` function.
4. Make a quick movie of the friends network over time using the `render.d3movie` function.

TERGM: Discrete time model

- Developed by Robins & Pattison (2001) and further developed by Hanneke et al. (2010)
- Scholars in political science most notably Cranmer and Desmarais (2011) have eased the use and highlighted the utility of these types of models for political science
- Extension of ERGM to the temporal setting is based on the idea of panel regression
- In a sequence of observations, lagged earlier observations or derived information thereof can be used as predictors for later observations.
    - In other words, some of the statistics are direct functions of an earlier realization of the network
    - In its most basic form, the TERGM is a conditional ERGM with an earlier observation of the network occurring among the predictors.

TERGM: Discrete time model

- To extend ERGM to a longitudinal context, Hanneke et al. (2010) make a Markov assumption on the network from one time step to the next
- Specifically, given an observed network Y^t, make the assumption that Y^t is independent of $Y^1, \ldots, Y^{t-2}$
- Thus a sequence of network observations has the property that:

$$Pr(Y^2, Y^3, \ldots, Y^t | Y^1) = Pr(Y^t | Y^{t-1}) Pr(Y^{t-1} | Y^{t-2}) \ldots Pr(Y^2 | Y^1)$$

TERGM: Discrete time model

- With this assumption in mind we just need to choose a form for the conditional PDF of $P(Y^t|Y^{t-1})$
- $Y^t|Y^{t-1}$ can be expressed through an ERGM distribution, which then gives us what is referred to as a TERGM:

$$\Pr(Y^t|\theta, Y^{t-1}) = \frac{\exp(\theta^T g(Y^t, Y^{t-1},))}{k}$$

TERGM: Block-diag visualization

- TERGM is essentially estimated through an ERGM with the dependent variable modeled as a block-diagonal matrix (such as below)
- Constraints are put on the model such that cross-network edges in the off-diagonal blocks are prohibited

	a_1	b_1	c_1	d_1	a_2	b_2	c_2	d_2	a_3	b_3	c_3	d_3
a_1	0	1	0	0	×	×	×	×	×	×	×	×
b_1	0	0	1	0	×	×	×	×	×	×	×	×
c_1	0	1	0	0	×	×	×	×	×	×	×	×
d_1	0	0	0	0	×	×	×	×	×	×	×	×
a_2	×	×	×	×	0	1	0	0	×	×	×	×
b_2	×	×	×	×	1	0	0	0	×	×	×	×
c_2	×	×	×	×	1	1	0	1	×	×	×	×
d_2	×	×	×	×	1	0	0	0	×	×	×	×
a_3	×	×	×	×	×	×	×	×	0	0	1	0
b_3	×	×	×	×	×	×	×	×	1	0	0	0
c_3	×	×	×	×	×	×	×	×	0	1	0	1
d_3	×	×	×	×	×	×	×	×	1	1	0	0

btergm Package

- The `btergm` package has been developed by Leifeld, Cranmer, & Desmarais (2018) to estimate longitudinal networks using TERGM

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Temporal Exponential Random Graph Models with btergm: Estimation and Bootstrap Confidence Intervals

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- Package provides two functions to estimate a TERGM, one using a pseudolikelihood (`btergm`) and the other using MCMC-MLE (`mtergm`)

Running a TERGM

- We are going to run a TERGM on the longitudinal alliance network, and will employ the following specification:
  - `edges`: density term
  - `edgecov(war)`: list of matrices where cross-sections denote war
  - `edgecov(contiguity)`: matrix of distances between countries
  - `absdiff(polity)`: Absolute difference between polity of i and j
  - `absdiff(cinc)`: Absolute difference between cinc of i and j
  - `gwesp(.5, fixed = TRUE)`: Geometric weighted triangle term

Running a TERGM

```
library(btergm)

tergmFit <- btergm(
    ally ~ edges +
    edgecov(war) + edgecov(contiguity) +
    nodecov('polity') + absdiff("polity") +
    nodecov('cinc') + absdiff("cinc") +
    gwesp(.5, fixed = TRUE)
    )
```

```
##                   t=1 t=2 t=3 t=4 t=5 t=6 t=7 t=8 t=9 t=10
## ally (row)         50  50  50  50  50  50  50  50  50   50
## ally (col)         50  50  50  50  50  50  50  50  50   50
## war (row)          50  50  50  50  50  50  50  50  50   50
## war (col)          50  50  50  50  50  50  50  50  50   50
## contiguity (row)   50  50  50  50  50  50  50  50  50   50
## contiguity (col)   50  50  50  50  50  50  50  50  50   50
##                                 t=1 t=2 t=3 t=4 t=5 t=6 t=7 t=8 t=9 t=10
## maximum deleted nodes (row)       0   0   0   0   0   0   0   0   0    0
## maximum deleted nodes (col)       0   0   0   0   0   0   0   0   0    0
## remaining rows                   50  50  50  50  50  50  50  50  50   50
## remaining columns                50  50  50  50  50  50  50  50  50   50
##                   t=1 t=2 t=3 t=4 t=5 t=6 t=7 t=8 t=9 t=10
```

Results

```
summary(tergmFit)
```

```
##                              Estimate   Boot mean    2.5%   97.5%
## edges                     -5.69586158 -5.69973953 -5.8424 -5.4809
## edgecov.war[[i]]          -0.07515258 -0.07292465 -0.3130  0.1482
## edgecov.contiguity[[i]]    2.54540447  2.54635512  2.4943  2.6080
## nodecov.polity            -0.00027339 -0.00027453 -0.0064  0.0057
## absdiff.polity             0.01865488  0.01875071  0.0143  0.0235
## nodecov.cinc              -1.42023487 -1.43086995 -2.1052 -0.8640
## absdiff.cinc              26.96999913 27.00237695 26.1670 27.7765
## gwesp.fixed.0.5            2.24553096  2.24742783  2.1249  2.3382
```

Duque (2018)

The DV is diplomatic ties, `dipl_ties`:

```
#Clear your memory and unload `btergm` as it clashes with `network`:

detach("package:btergm", unload=TRUE)
data("duqueData")

class(dipl_ties)
```

```
## [1] "list"
```

```
length(dipl_ties)
```

```
## [1] 8
```

```
class(dipl_ties[[1]])
```

```
## [1] "data.frame"
```

The Dependent Variable

we can see that `dipl_ties` is currently a list of `data.frames`. Let's convert it into a list of networks.

```
library(statnet)
for (i in 1:8) {
dipl_ties[[i]]<-as.network(as.matrix(dipl_ties[[i]]))
}
class(dipl_ties[[1]])
```

```
## [1] "network"
```

Use `networkDynamic` for Visualizing the Networkrk

```
diplDyn = networkDynamic(network.list=dipl_ties, vertex.pid='vertex.n
```

Get an error that need vertex.pid (persistent identifiers), as our networks are not of equal size.

Try Again

```
#Define network pids:
for (i in 1:8) {
set.network.attribute(dipl_ties[[i]], 'vertex.pid','vertex.names')
}
#Takes 5 min to run:
diplDyn = networkDynamic(network.list=dipl_ties, vertex.pid='vertex.n
diplDyn
```

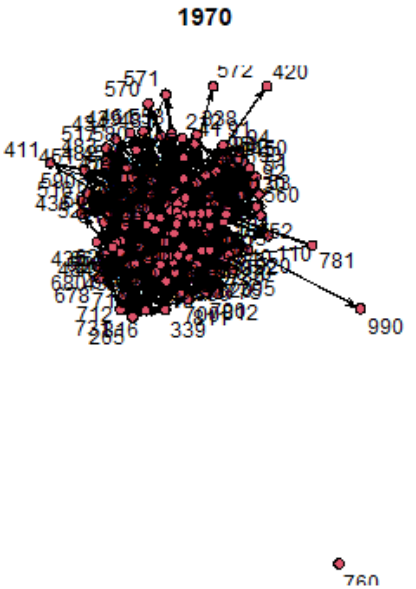
```
## NetworkDynamic properties:
##   distinct change times: 9
##   maximal time range: 0 until  8
##
## Includes optional net.obs.period attribute:
##  Network observation period info:
##   Number of observation spells: 1
##   Maximal time range observed: 0 until 8
##   Temporal mode: discrete
##   Time unit: step
##   Suggested time increment: 1
##
##  Network attributes:
##   vertices = 194
##   directed = TRUE
```

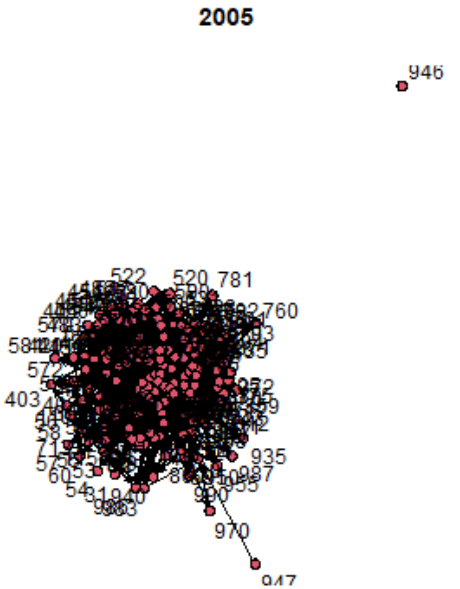
Quick snapshots

```
par(mfrow = c(1,2))
plot(
    network.extract(diplDyn, at = 1),
    main = "1970", displaylabels = T)
plot(
    network.extract(diplDyn, at = 6),
    main = "2005", displaylabels = T)
```

Note: cannot use coordinates, because networks are not of equal size, include different actors.

Quick snapshots





What Have We Learned?

- Networks of embassies are dense.
- Some states host very few embassies.
- Note: network graphs of dense networks are not very esthetically pleasing or informative.

Descriptive analyses

`tsna` enables us to quickly calculate some basic descriptive statistics such as the density of a graph:

```
library(tsna)
tSnaStats(diplDyn, "gden") # Changes in graph density
```

```
## Time Series:
## Start = 0
## End = 8
## Frequency = 1
##         Series 1
##  [1,] 0.2050836
##  [2,] 0.2368542
##  [3,] 0.2169688
##  [4,] 0.2145924
##  [5,] 0.2195870
##  [6,] 0.1851018
##  [7,] 0.1902708
##  [8,] 0.2099215
##  [9,]        NA
```

Descriptive analyses

Can also examine transitivity over time:

```
tSnaStats(diplDyn, "gtrans") # Changes in graph transitivity
```

```
## Time Series:
## Start = 0
## End = 8
## Frequency = 1
##         Series 1
##  [1,] 0.4830388
##  [2,] 0.4953440
##  [3,] 0.5264074
##  [4,] 0.5159558
##  [5,] 0.5269708
##  [6,] 0.4841114
##  [7,] 0.4875437
##  [8,] 0.5132955
##  [9,]        NA
```

Descriptive analyses

The `tErgmStats` enables us to calculate changes in `ergm` terms over time:

```
tErgmStats(diplDyn, "~ edges+triangle")
```

```
## Time Series:
## Start = 0
## End = 8
## Frequency = 1
##   edges triangle
## 0  3655   100235
## 1  5153   190717
## 2  5314   203523
## 3  5597   215072
## 4  5870   238736
## 5  6165   236593
## 6  6618   270378
## 7  7380   336926
## 8     0        0
```

Duque (2018)

Popularity hypothesis: High-status states should receive more recognition simply because of their position in the social structure, rather than because of the possession of status attributes (`2-instars`).

Reciprocity and transitivity: A state's existing relations should influence the state's ability to achieve status (`mutual` and `triangle`).

Homophily: States should recognize states that have similar values and resources as them (`absdiff`).

Dyadic Covariates

Contiguity (`contig`) and alliances (`allies`) are time-varying edge-level covariates. We must make sure that they are stored as lists of matrices.

```
#Contiguity:
class(contig)
```

```
## [1] "list"
```

```
length(contig)
```

```
## [1] 8
```

```
class(contig[[1]])
```

```
## [1] "data.frame"
```

```
dim(contig[[1]])
```

```
## [1] 134 134
```

```
contig[[1]][1:3,1:3]
```

```
##    2 20 40
## 2  0  1  0
## 20 1  0  0
## 40 0  0  0
```

Allies

```
#Allies:
class(allies)
```

```
## [1] "list"
```

```
length(allies)
```

```
## [1] 8
```

```
class(allies[[1]])
```

```
## [1] "data.frame"
```

```
dim(allies[[1]])
```

```
## [1] 134 134
```

```
allies[[1]][1:3,1:3]
```

```
##    2 20 40
## 2  0  1  0
## 20 1  0  0
## 40 0  0  0
```

Dyadic Covariates

It looks like `allies` and `contig` are currently stored as lists of `data.frames`. We must convert them to lists of matrices.

```
for (i in 1:8) {
contig[[i]]<-as.matrix(contig[[i]])
allies[[i]]<-as.matrix(allies[[i]])
}
```

Now set up DV with vertex and dyadic attributes

Our node-level covariate, `polity$dem_dum` must be defined as a vertex attribute in each of the `dipl_ties` networks.

```
#Define Dem as a vertex attribute for each year of dipl_ties (didn't
  set.vertex.attribute(dipl_ties[[1]],"dem",polity$dem_dum[polity$yea
  set.vertex.attribute(dipl_ties[[2]],"dem",polity$dem_dum[polity$yea
  set.vertex.attribute(dipl_ties[[3]],"dem",polity$dem_dum[polity$yea
  set.vertex.attribute(dipl_ties[[4]],"dem",polity$dem_dum[polity$yea
  set.vertex.attribute(dipl_ties[[5]],"dem",polity$dem_dum[polity$yea
  set.vertex.attribute(dipl_ties[[6]],"dem",polity$dem_dum[polity$yea
  set.vertex.attribute(dipl_ties[[7]],"dem",polity$dem_dum[polity$yea
  set.vertex.attribute(dipl_ties[[8]],"dem",polity$dem_dum[polity$yea

dipl_ties[[1]] %v% "dem"
```

```
##   [1] 1 1 0 0 0 1 0 0 0 0 0 0 0 1 0 1 1 0 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 0
##  [38] 0 1 0 0 1 0 0 0 0 1 0 0 0 1 1 1 1 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##  [75] 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 1 0 0 0 0 1 0 0 0 0
## [112] 0 0 0 1 1 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 1 1 0
```

```
dipl_ties[[2]] %v% "dem"
```

Specify the Model:

```
library(btergm)
#This runs for 5 min:
tergm_Duque<-btergm(dipl_ties ~ edges + istar(2) + ostar(2) + mutual
                    absdiff("dem")+
                    +nodeicov("dem")+
                    +edgecov(allies)
                    +edgecov(contig),
                    R=1000)

summary(tergm_Duque)
```

Run the Model

```
##                          Estimate Boot mean    2.5%   97.5%
## edges                  -5.377882 -5.405644 -5.6133 -5.1253
## istar2                  0.028279  0.028712  0.0267  0.0313
## ostar2                  0.029487  0.030022  0.0266  0.0340
## mutual                  2.485567  2.482407  2.3500  2.5956
## triangle                0.011750  0.011628  0.0103  0.0127
## absdiff.dem            -0.287938 -0.289257 -0.3656 -0.2207
## nodeicov.dem           -0.204423 -0.195365 -0.2515 -0.1246
## edgecov.allies[[i]]     1.205762  1.220685  1.1238  1.3205
## edgecov.contig[[i]]     1.765076  1.765720  1.5732  2.0623
```

Results

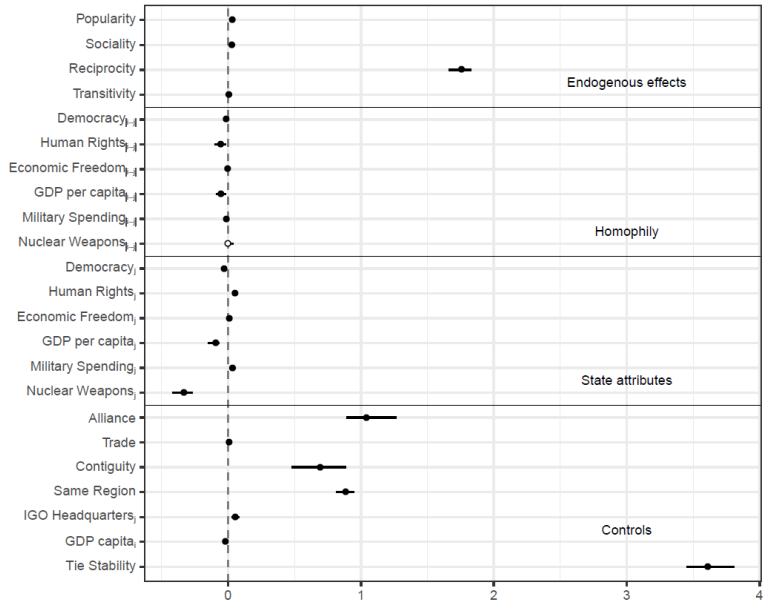


Figure 3. Temporal exponential random graph model of diplomatic ties for 1995-2005.

Temporal Terms

- `delrecip(mutuality = FALSE, lag = 1)`: checks for delayed reciprocity. For example, if node j is tied to node i at $t = 1$, does this lead to a reciprocation of that tie back from i to j at $t = 2$? If mutuality = TRUE is set, this extends not only to ties, but also non-ties. The lag argument controls the size of the temporal lag: with $lag = 1$, reciprocity over one consecutive time period is checked. Note that as lag increases, the number of time steps on the dependent variable decreases.

- `memory(type = "stability", lag = 1)`: controls for the impact of a previous network on the current network. Four different types of memory terms are available: positive autoregression (type = "autoregression") checks whether previous ties are carried over to the current network; dyadic stability (type = "stability") checks whether both edges and non-edges are stable between the previous and the current network; edge loss (type = "loss") checks whether ties in the previous network have been dissolved and no longer exist in the current network; and edge innovation (type = "innovation") checks whether previously unconnected nodes have the tendency to become tied in the current network.

Temporal Terms Cont'd

- `timecov(x = NULL, minimum = 1, maximum = NULL, transform = function(t) t)`: checks for linear or non-linear time trends with regard to edge formation. Optionally, this can be combined with a covariate to create an interaction effect between a dyadic covariate and time in order to test whether the importance of a covariate increases or decreases over time. In the default case, edges modeled as being linearly increasingly important over time. By tweaking the transform function, arbitrary functional forms of time can be tested. For example, transform = sqrt (for a geometrically decreasing time effect), transform = function(x) x^2 (for a geometrically increasing time effect), transform = function(t) t (for a linear time trend) or polynomial functional forms (e.g., 0 + (1 *t)* + *(1* t^2)) can be used.

Example with Time Vars

```
#This runs for 5 min:
tergm_Duque1<-btergm(dipl_ties ~ edges + istar(2) + ostar(2) + mutual
                   absdiff("dem")+
                   +nodeicov("dem")+
                   +edgecov(allies)+
                   +edgecov(contig)+
                   timecov(),
                   R=1000)

summary(tergm_Duque1)
```

Example with Time Vars

```
##                          Estimate Boot mean    2.5%   97.5%
## edges                   -4.915580 -4.944023 -5.2673 -4.5906
## istar2                   0.029973  0.030279  0.0283  0.0333
## ostar2                   0.031485  0.031775  0.0289  0.0353
## mutual                   2.428539  2.424032  2.2767  2.5464
## triangle                 0.011518  0.011492  0.0102  0.0126
## absdiff.dem             -0.298055 -0.300189 -0.3669 -0.2392
## nodeicov.dem            -0.122282 -0.130947 -0.2013 -0.0506
## edgecov.allies[[i]]      1.199983  1.217104  1.0998  1.3312
## edgecov.contig[[i]]      1.837357  1.828172  1.6100  2.1092
## edgecov.timecov1[[i]] -0.126189 -0.127274 -0.2050 -0.0802
```

Your Turn

Re-specify the model to account for delayed reciprocity and stability. Estimate your new model.

Your Turn 1

1. Set up `smoke` and `drink` as vertex attributes to the friendship network.

2. Estimate a temporal `ergm` that models friendships as a function of

    - drinking and smoking behaviors,
    - homophily (people make friends with those with similar drinking and smoking habits),
    - reciprocity (why?)
    - triad closure (why?)
    - popularity

3. Add temporal terms to model stability and delayed reciprocity.

Bayesing ERGM (BERGM)

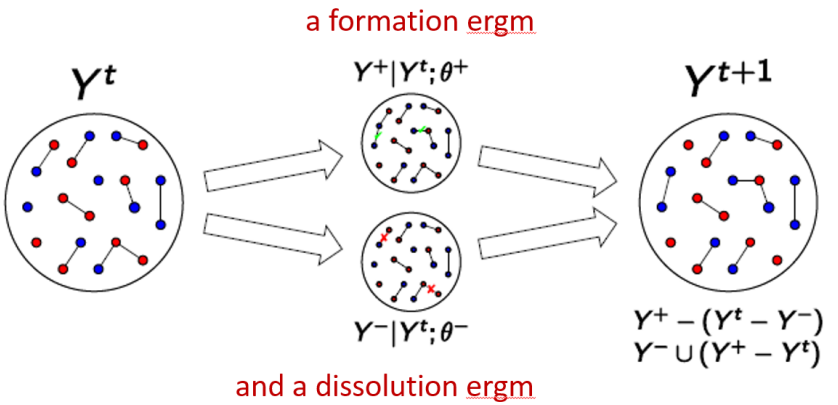
- Paper: Caimo & Friel (2011)
- `Bergm` on CRAN
- Vignette

ego-ERGM

- Paper: Salter-Townshend & Murphy (2016)

Separable temporal ERGM (STERGM)

- Paper: Krivitsky & Handcock (2012)
- `tergm` on CRAN
- Vignette



Hierarchical Exponential-Family Graph Model (HERGM)

- Paper: Schweinberger & Handcock (2015)
- `hergm` on CRAN
- Vignette

Multilevel ERGM

- Paper: Wang et al. (2016)

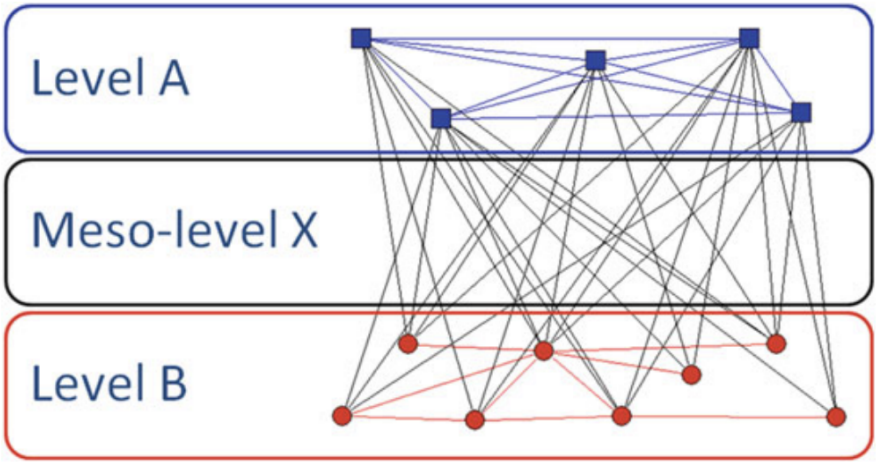


Fig. 6.1 A two-level network representation