Comparison of Latent Variable Models for Network Analysis

Types of Dependencies

We have explored four types of latent variable models: the Social Relations Model (SRM), Latent Distance Model (LDM), Latent Block Model (LBM), and Latent Factor Model (LFM). These models differ in their capacity to account for various orders of dependencies within network structures. The Social Relations Model is set up only to capture first-order (actor-level) effects and second-order (reciprocity) effects, but it can be extended to a variety of network types easily.

While the LDM doesn't explicitly model first and second order dependencies, these characteristics are implicitly accounted for by the model's placement of actors into a latent Euclidean social space. Nonetheless, nodal random effects and reciprocity can be directly added to the model to explicitly account for first and second order dependencies. The LDM's primary strength lies in capturing homophily, a key third order dependency, although it falls short in characterizing stochastic equivalence.

The block model takes a distinct approach, primarily focusing on capturing stochastic equivalence by grouping actors into blocks. It's worth noting that the block calculation itself doesn't guarantee accounting for first or second order dependencies.

Among the four, the Latent Factor Model stands out as the most versatile, capable of capturing all types of dependencies. Its ability to model first-order, second-order, homophily, and stochastic equivalence effects makes it an adaptable choice for complex network structures.

Types of Dependencies	Social Relations Model	Latent Distance Model	Latent Block Model	Latent Factor Model
First Order (Actor Level)	+	+		+
Second Order (Reciprocity)	+	+		+
Third Order - Homophily		+		+

Types of Dependencies	Social Relations Model	Latent Distance Model	Latent Block Model	Latent Factor Model
Third Order - Stochastic Equivalence			+	+

An important point to emphasize is that the social relations model is rarely employed in isolation. It's typically paired with either the latent distance or factor models.

Most typically, however the SRM and LFM are used together in a model called AMEN. This model is a combination of the SRM and LFM, and it allows for the estimation of both additive and multiplicative effects.

Modeling Features

This section's table compares practical aspects of using these models, highlighting their strengths and limitations in various analytical contexts.

In theory, all four models can accommodate different types of network data, be it weighted/unweighted or symmetric/asymmetric. Each model also allows for the estimation of exogenous covariate effects, enabling researchers to control for or investigate actor or dyad-level variables. The latent block model is a bit of an exception here mostly because the implementations of it in software are often focused more on just generating the blocks and there is not as much interest in incorporating covariates. The NetMix package here is somewhat of an exception and Tracy Sweet, a statistician at University of Maryland, have made good progress on this dimension but it is still a work in progress (NetMix only works on binary outcomes and Sweet's code covers select types of networks).

Regarding interpretability, the Social Relations Model and Latent Block Model generally offer more intuitive results. The SRM's key random effects indicate how much more or less active an actor is after accounting for exogenous covariates. Block models are similarly straightforward to interpret, as they essentially categorize actors into different blocks, allowing users to easily determine the various roles nodes play in a network based on their block classification.

The Latent Distance Model is also considered relatively interpretable because it places actors in a Euclidean social space, where an actor's position alone determines their tie probability. This works exceptionally well when the social space is two-dimensional, but interpretation becomes more challenging with three or more dimensions.

Despite its high flexibility, the Latent Factor Model can be the most challenging to interpret due to its complexity. Instead of projecting actors into classes or a Euclidean social space, it places them in a vector space, which can be difficult to interpret.

Computation time varies across models. The Social Relations Model and Latent Block Model typically compute faster, especially for large networks. In contrast, the Latent Distance Model and Latent Factor Model can be more computationally intensive. It's important to note that none of these models are inherently designed for causal inference or directly estimating transitivity effects, which remain challenging areas in network analysis. The choice between these models often depends on the specific research question, the features of the network being studied, and the available computational resources.

Modeling Features	Social Relations Model	Latent Distance Model	Latent Block Model	Latent Factor Model
Accomodates Various Distribution Types	+	+	+	+
Accomodates Covariates	+	+	~	+
Ease of Interpretability	+	~	+	
Computation Time	+		+	
Useful for Causal Inference				
Estimate the Direct Effect of Transitivity (we'll need graph-based models such as ERGM and SAOM for this)				

Software Packages for Model Estimation

Based on the lecture materials, here are the software packages that can be used to estimate each of these models:

1. Social Relations Model (SRM):
    - The 'amen' package in R can be used to estimate the SRM. This package, developed by Hoff et al., provides a flexible framework for fitting additive and multiplicative effects models for relational data.

- The `latentnet` package in R, part of the statnet suite, can also be used to estimate a version of the SRM.
- It's worth reiterating that scholars rarely use the SRM in isolation; it's typically paired with the LDM or LFM.

2. Latent Distance Model (LDM):
   - 'latentnet' is the go-to package for estimating this model.
   - The 'VBLPCM' package in R (Salter-Townshend, 2015) offers tools for variational Bayesian inference for latent position and cluster models.
3. Latent Block Model (LBM):
   - The 'blockmodels' package in R (Leger, 2015) can estimate stochastic block models from weighted and unweighted networks, though it doesn't accommodate covariates.
   - The 'sbm' package in R (Chiquet et al., 2021) provides tools for stochastic blockmodeling of various network types and can handle covariates. Nonetheless, successfully estimating models with covariates can be challenging.
   - Another notable option is the 'NetMix' package (Olivella et al., 2021). `NetMix` boasts fast estimation schemes and can accommodate covariates and longitudinal networks. At present, it only supports binary networks.
4. Latent Factor Model (LFM):
   - The 'amen' package in R can also estimate Latent Factor Models. It offers a unified framework for additive and multiplicative effects network models.

Related Papers

1. Social Relations Model:
   - Hoff (2005)
     - Technical paper by Peter introducing the method along with an earlier version of the LFM.
   - Hoff (2018a)
     - Vignette from the `amen` package in R (note that AMEN is a combination of the SRM + LFM).
   - Hoff (2018b)
     - Technical paper detailing all the models implemented in the `amen` package.
   - Dorff & Ward (2013)
     - Showcases how SRM can be used in political science data broadly.
   - Dorff & Minhas (2016)
     - Application of SRM in a measurement context to the sanctions network.
   - Minhas et al (2021)

    - Meant to be a friendly technical discussion of the various parts of AMEN (remember AMEN = LFM + SRM) and provides simulations to showcase inferential properties and applications.
2. Latent Distance Model:
    - Hoff et al (2002)
        - The original paper introducing the LDM.
    - Krivitsky & Handcock (2008)
        - A paper introducing the `latentnet` package in R.
    - Sewell & Chen (2015)
        - A notable paper that talks about how the LDM can be extended to a longitudinal context.
    - Kirkland (2012)
        - Application of LDM to American politics data.
    - Berlusconi et al (2017)
        - Fun application to modeling heroin flows.
3. Latent Block Model:
    - Leger (2016)
        - A paper introducing the `blockmodels` package in R.
    - Chiquet et al (2024)
        - Website for `sbm` package.
    - Olivella et al (2021)
        - GitHub repository for the `NetMix` package.
4. Latent Factor Model:
    - Again Hoff (2005), Hoff (2018a), and Hoff (2018b) are the papers where the key parts of the model were originally developed. Minhas et al (2021) is again useful here and comes with a lot of interpretation guidance and some simulations to study strengths and weaknesses of the model.
    - Weschle (2018)
        - Uses LFM to measuring political relationships.
    - Dorff et al (2020)
        - Application to a longitudinal intrastate conflict network and extends the model to handle changing actor composition.
    - Cheng & Minhas (2020)
        - Uses LFM as a measurement model to measure state preferences for an application to understanding foreign aid responses to natural disasters.
    - Huhe et al (2021)
        - Uses LFM as a measurement model to measure factions in China CCP.
5. Other:

    - Kim et al (2018)
        - Review piece discussing the history of these various approaches.
    - Minhas et al (2018)
        - Compares various inferential models for networks.
6. Causal Approaches to Networks (Elizabeth Ogburn is someone who is doing really cool work here):
    - An et al (2022)
        - Review piece on causal inference in networks.
    - Lee & Ogburn (2020)
        - No real new method here just a call to the problem.
    - Ogburn et al (2022)
        - Discusses how to handle causal inference in cross-sectional networks.
    - Aronow et al (2017)
        - Not about causal inference explicitly but deals with how to correct for standard errors when first and second order dependence are suspected.
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